Block Cipher

The Workhorse of Crypto

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Block Cipher

- \triangleright Block ciphers take as input a block of plaintext and generate the corresponding ciphertext.
- \blacktriangleright Fundamental goal of a block cipher is to provide data confidentiality.
- \blacktriangleright They also serve as a primary building block for other cryptographic primitives like MAC, hash functions etc.

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- \triangleright Symmetric key block ciphers are extremely fast.
- \blacktriangleright Example:
	- 1. Data Encryption Standard.
	- 2. Advanced Encryption Standard.

Definition

- A deterministic cryptosystem $\mathcal{E} = (E; D)$
	- \triangleright Message space and ciphertext space: a finite set X
	- Exergence: K .
	- \triangleright \mathcal{E} is a block cipher defined over $(\mathcal{K}; \mathcal{X})$.
- For every key $k \in \mathcal{K}$, define $f_k := E(k;)$

$$
f_k:\mathcal{X}\longrightarrow\mathcal{X}
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- For correctness of decryption what property f_k needs?
	- \blacktriangleright f_k must be a permutation on X
	- ▶ $D(k;)$ is the inverse function f_k^{-1} .

\triangleright What security property should $\mathcal E$ satisfy?

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- \triangleright Suppose: block-size = key-size = 128-bits
	- \blacktriangleright How many permutation functions are possible?
	- \blacktriangleright How many f_k are possible?
- \blacktriangleright What security property should $\mathcal E$ satisfy?
- \triangleright Computationally indistinguishable from a random permutation.
- \triangleright Suppose: block-size $=$ key-size $=$ 128-bits
	- \blacktriangleright How many permutation functions are possible?
	- \blacktriangleright How many f_k are possible?
- \blacktriangleright Claim:
	- \blacktriangleright A secure block cipher is unpredictable.
	- \blacktriangleright Unpredictability implies key recovery is infeasible.

Block Cipher: Desirable Properties

\blacktriangleright Security:

- 1. Confusion: relationship between the key and the ciphertext should be complicated.
- 2. Diffusion: every single ciphertext bit should depend on all the plaintext bits.
- 3. Keysize: small enough to manage but large to make exhaustive search infeasible.

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- \blacktriangleright Efficiency:
	- 1. Encryption and decryption rate should be high.
	- 2. Easy to implement (and analyze).
	- 3. Suitable for hardware and/or software.

Data Encryption Standard (DES)

- \blacktriangleright The first commercially available modern cipher with fully specified implementation details in the open literature.
	- \blacktriangleright Note: Design principles are still classified.
- In the early 70s US National Bureau of Standards (now $NIST$) solicited proposals for encryption algorithms to protect computer data.
- \blacktriangleright IBM's submission was later adopted as DES.
- In the early eighties DES was adopted as a US Banking Standard and used widely all over the world.
- \triangleright DES has a block size of 64-bits and key size of 56-bits.
- \triangleright NSA (allegedly) forced the keysize to be restricted to 56 -bits.
	- In 1977 Diffie and Hellman suggested that a special purpose machine can be built to exhaustively search the keyspace of DES at an estimated cost of USD 20M.

Feistel Network/Cipher

Parameters:

1. Block length: $2n$ -bits (divided into two equal halves).

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- 2. Key size: ℓ -bits.
- 3. Number of rounds: r.

$$
\blacktriangleright \mathcal{M} = ?, \mathcal{C} = ? \text{ and } \mathcal{K} = ?
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Feistel Network/Cipher

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- 1. Block length: $2n$ -bits (divided into two equal halves).
- 2. Key size: ℓ -bits.
- 3. Number of rounds: r.
- $M = ?$, $C = ?$ and $K = ?$
- Exey Scheduling Algorithm: Derive ℓ' -bit "subkeys" k_1, k_2, \ldots, k_r from the secret key k .
- \triangleright Round function: Each subkey defines a round function:

$$
f_i: \{0,1\}^n \times \{0,1\}^{\ell'} \rightarrow \{0,1\}^n
$$

 \blacktriangleright Such a block cipher is called iterated block cipher.

Encryption/Decryption

Encryption proceeds through r rounds.

- Divide the 2*n*-bit message into two equal halves: $m = (m_0, m_1)$.
- ▶ Round 1: (m_0, m_1) → (m_1, m_2) where $m_2 = m_0 \oplus f_1(m_1, k_1)$.
- ▶ Round 2: (m_1, m_2) \rightarrow (m_2, m_3) where $m_3 = m_1 \oplus f_2(m_2, k_2)$. \blacktriangleright \ldots
- ► Round r: $(m_{r-1}, m_r) \rightarrow (m_r, m_{r+1})$ where $m_{r+1} = m_{r-1} \oplus f_r(m_r, k_r)$.

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 \triangleright Ciphertext: $c = (m_{r+1}, m_r)$.

Decryption: ?

Encryption/Decryption

Encryption proceeds through r rounds.

- **Divide the 2n-bit message into two equal halves:** $m = (m_0, m_1)$.
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- ► Round r: $(m_{r-1}, m_r) \rightarrow (m_r, m_{r+1})$ where $m_{r+1} = m_{r-1} \oplus f_r(m_r, k_r)$.
- \triangleright Ciphertext: $c = (m_{r+1}, m_r)$.

Decryption: ?

Same process with keys reversed!

- ► Given $c = (m_{r+1}, m_r)$, compute $m_{r-1} = m_{r+1} \oplus f_r(m_r, k_r)$.
- **IF** Then compute $m_{r-2}, \ldots, m_1, m_0$.

\blacktriangleright The encryption must be invertible.

The round function f_i must be invertible. [True/False?]

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- \blacktriangleright The encryption must be invertible.
	- \blacktriangleright The round function f_i must be invertible. [True/False?]
- \blacktriangleright Implementation:
	- \triangleright Encryption: Implement just one round and then reuse the code for the other rounds.

- \triangleright Decryption: The same code for encryption can be reused with the subkeys used in reverse order.
- \triangleright DES is an example of Feistel cipher with $n = 32$, $r = 16$ and $\ell = 56$.
- Exhaustive key search requires only 2^{56} steps and can be easily parallelized.
- \blacktriangleright DES Challenge from RSA Security: given three pairs of (m, c) find the corresponding key.
	- 1. [1997] The first challenge was broken in 96 days.
	- 2. [1998] The second challenge was broken in 56 hours by Deep Crack machine of Electronic Frontier Foundation (EFF).
	- 3. [1999] The third challenge was broken in 22 hours 15 minutes by Deep Crack and a network of around 100,000 computers.
- \triangleright See www.distributed.net if you're interested!
- \triangleright DES is a deterministic encryption scheme.
	- \triangleright Same message encrypted under the same key always gives the same ciphertext.
- If plaintext blocks are distributed uniformly at random then we can expect a collision with a high probability after observing 2^{32} ciphertext blocks.
	- \triangleright Ciphertext reveals some information about the underlying plaintext.

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 \blacktriangleright Re-encrypt the ciphertext once (or more) using independent keys.

- \triangleright Double-DES:
	- **I** Key: (k_1, k_2) .
	- Encryption: $E_{k_2}(E_{k_1}(m))$, E is DES.
	- Exercise Key length is now double $(112-bits)$ but block length remains 64-bits.

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- \triangleright Does multiple encryption always give increased security?
- ► Fix a DES key k , E_k : $\{0,1\}^{64}$ \rightarrow $\{0,1\}^{64}$ defines a permutation.
- The 2^{56} keys define 2^{56} such potentially different permutations.
- I What if given any two k_1, k_2 there exists a k_3 s.t $E_{k_3}(m) = E_{k_2}(E_{k_1}(m))$?

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- I What if given any two k_1, k_2 there exists a k_3 s.t $E_{k_3}(m) = E_{k_2}(E_{k_1}(m))$?
- **Fact:** The set of 2^{56} permutations defined by 2^{56} DES keys is not closed under functional composition.

Double DES

- **I** Key: (k_1, k_2) .
- Encryption: $c = E_{k_2}(E_{k_1}(m))$, E is DES encryption.
- Decryption: $m = E_{k_1}^{-1}$ $\frac{1}{k_1}(\bar{E}^{-1}_{k_2})$ $\frac{(-1)}{k_2}(c)$
- \triangleright Key length is now double $(112-bits)$ exhaustive key search is infeasible.

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 \triangleright Block length remains 64 -bits.

Note: DES is an endomorphic cryptosystem, $P = C$.

Is Double-DES more secure than DES?

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 $\mathsf{c} = \mathsf{E}_{k_2}(\mathsf{E}_{k_1}(m))$ thus $\mathsf{E}_{k_2}^{-1}$ $E_{k_2}^{-1}(c) = E_{k_1}(m).$

 $\mathsf{c} = \mathsf{E}_{k_2}(\mathsf{E}_{k_1}(m))$ thus $\mathsf{E}_{k_2}^{-1}$ $E_{k_2}^{-1}(c) = E_{k_1}(m).$ Input: 3 known plaintext/ciphertext pairs $(m_1, c_1), (m_2, c_2), (m_3, c_3)$. Output: The secret key (k_1, k_2) .

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Input: 3 known plaintext/ciphertext pairs $(m_1, c_1), (m_2, c_2), (m_3, c_3)$. Output: The secret key (k_1, k_2) .

- 1. For each $h_2 \in \{0,1\}^{56}$:
	- ▶ Compute $E_{h_2}^{-1}(c_1)$ and store $[E_{h_2}^{-1}(c_1), h_2]$ in a table $\mathbb T$ sorted by the first component.

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- 1. For each $h_2 \in \{0,1\}^{56}$:
	- ▶ Compute $E_{h_2}^{-1}(c_1)$ and store $[E_{h_2}^{-1}(c_1), h_2]$ in a table $\mathbb T$ sorted by the first component.
- 2. For each $h_1 \in \{0,1\}^{56}$ do the following:
	- 2.1 Compute $E_{h_1}(m_1)$
	- 2.2 Search for $E_{h_1}(m_1)$ in \mathbb{T} $(E_{h_1}(m_1)$ matches table entry $[E_{h_2}^{-1}(c_1), h_2]$ if $E_{h_1}(m_1) = E_{h_2}^{-1}(c_1)$.
	- 2.3 For each match $[E_{h_2}^{-1}(c_1), h_2]$ in the table, check whether $E_{h_2}(E_{h_1}(m_2))=c_2$; if so then check whether $E_{h_2}(E_{h_1}(m_3))=c_3.$
	- 2.4 If both checks pass, then output (h_1, h_2) and STOP.

Why 3 Plaintext/Ciphertext Pairs?

- Suppose E is a block cipher with key space $\mathcal{K} = \{0, 1\}^{\ell}$, and plaintext/ciphertext space $\mathcal{P} = \mathcal{C} = \{0, 1\}^n$.
- ▶ Suppose $k' \in \mathcal{K}$ is the secret key and (m_i, c_i) , $1 \le i \le t$ are the known plaintext/ciphertext pairs, where m_i s are all distinct.

 \blacktriangleright $c_i = E_{k'}(m_i)$ for all $1 \leq i \leq t$.

 \triangleright What should be the value of t to ensure (with very high probability) that there is only one key $k'\in\mathcal{K}$ such that $E_{k'}(m_i)=c_i$ for all $1 \le i \le t$?

Why 3-Pairs of PT/CT is Enough

For each $k \in \mathcal{K}$, $E_k : \{0,1\}^n \to \{0,1\}^n$ is a permutation.

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- ► [Heuristic Assumption] For each $k \in \mathcal{K}$, E_k is a random function (i.e., a randomly selected function).
	- 1. The assumption is not correct as E_k is not random and a random function is almost certainly not a permutation.
	- 2. However, the assumption turns out to be quite good for the analysis!

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	- 1. The assumption is not correct as E_k is not random and a random function is almost certainly not a permutation.
	- 2. However, the assumption turns out to be quite good for the analysis!
- Fix any $k \in \mathcal{K}$ s.t. $k \neq k'$ (k' is the unknown key we want).
- Probability that $E_k(m_i) = c_i$ for all $1 \le i \le t$ is

$$
\frac{1}{2^n}\cdot\frac{1}{2^n}\cdots\frac{1}{2^n}=\frac{1}{2^{nt}}
$$

The expected number of keys $k \in \mathcal{K}$ (excluding k') so that $E_k(m_i) = c_i$ for all $1 \leq i \leq t$ is:

$$
E_{\mathcal{K}} = \frac{2^{\ell} - 1}{2^{nt}}
$$

Analysis of Meet-in-the-Middle Attack

Double-DES Encryption: $c = E_{k_2}(E_{k_1}(m))$. Goal: Find (k_1, k_2) . Here $\ell = 112$ -bits and $n = 64$ -bits.

- 1. For $t=1, E_K \approx 2^{48}$.
	- Expected number of Double-DES keys (h_1, h_2) s.t. $E_{h_2}(E_{h_1}(m_1)) = c_1$ is 2^{48} .

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2. Among these 2^{48} keys, the expected number of keys that also satisfy $E_{h_2}(E_{h_1}(m_2)) = c_2$ is approximately $\frac{2^{48}}{2^{64}}$ $rac{2^{48}}{2^{64}} = \frac{1}{2^1}$ $rac{1}{2^{16}}$.

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- 2. Among these 2^{48} keys, the expected number of keys that also satisfy $E_{h_2}(E_{h_1}(m_2)) = c_2$ is approximately $\frac{2^{48}}{2^{64}}$ $rac{2^{48}}{2^{64}} = \frac{1}{2^1}$ $rac{1}{2^{16}}$.
- 3. For $t=$ 3, we have $E_{\cal K}\approx \frac{1}{2^8}$ $\frac{1}{2^{80}}$. If we find (h_1, h_2) s.t. $\mathcal{E}_{h_2}(\mathcal{E}_{h_1}(m_i)) = c_i$ for $1 \leq i \leq 3$ then with a very high probability $(h_1, h_2) = (k_1, k_2).$
- ▶ Number of DES evaluation: $2^{56} + 2^{56} + 2 \times 2^{48} \approx 2^{57}$.
- Storage requirement: $2^{56}(64 + 56)$ bits $\approx 10^6$ TB.

Conclusion: The effective key-length for Double-DES is essentially same as DES.

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 \triangleright Double-DES is not significantly secure than DES.

Claim: The memory requirement in the attack can be reduced at the expense of time – Time Memory Trade-Off Attack: Time: 2^{56+t} steps, Memory: 2^{56-t} units, $i\leq t\leq 55.$ Ref: A Cryptanalytic Time-Memory Trade-Off by Martin Hellman.

Triple-DES

- ► Key: (k_1, k_2, k_3) , where $k_1, k_2, k_3 \in R \{0, 1\}^{56}$.
- Encryption: $c = E_{k_3}(E_{k_2}(E_{k_1}(m)))$ where E is the DES Encryption function.

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- Decryption: $m = E_{k_1}^{-1}$ $\frac{1}{k_1} (E_{k_2}^{-1}$ $\frac{-1}{k_2} (E_{k_3}^{-1}$ $\frac{(-1)}{k_3}(c))$).
- \triangleright Key length is 168-bits and Block length is 64-bits.

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- \blacktriangleright Key length is 168-bits and Block length is 64-bits.
- Dictionary Attack: Adversary stores a large table $(\leq 2^{64})$ of plaintext-ciphertext pair.
	- \triangleright Counter-measure: Change secret key periodically.
- \triangleright Meet-in-the-Middle Attack: Takes approx. 2^{112} steps. [Exercise]
- \triangleright Sweet32: Birthday attack demonstrated in 2016 exploiting the 64-bit block size.
- \triangleright Triple-DES is still in use though it is suggested to be replaced by AES.

The basic design principle:

- 1. Round Cipher: a simple block cipher (\hat{E}, \hat{D})
	- **►** DES: $\hat{E}_k(x||y) = (y||x \oplus f(k, y))$
	- \blacktriangleright This one round cipher is obviously insecure!
- 2. Key Expansion: Use a simple function to expand the key k to r round keys k_1, k_2, \ldots, k_r .
- \triangleright Challenge: Design a round cipher which is very fast and gives a secure block cipher within a few rounds.
- \triangleright A linear function cannot be used to get a secure block cipher.

Substitution-Permutation Network (SPN)

Iterated block cipher where each round consists of a substitution and a permutation.

- In The secret key k is used to derive round keys $k_1, k_2, \ldots, k_h, k_{h+1}$ (h: number of rounds).
	- In the *i*-th round, k_i is XOR-ed with the input before applying the substitution.
- The output of the last round is XOR-ed with k_{h+1} to generate the ciphertext.
	- \triangleright Prevents the adversary from attempting to decrypt the ciphertext by undoing the final substitution and permutation operations.

If Whitening: Internal state of the cipher is protected by k_1 and k_{h+1} .

SPN: Encryption/Decryption

Encryption:

 $A \leftarrow \text{plaintext}$ For $i = 1$ h do: $A \leftarrow A \oplus k_i$ $A \leftarrow S(A)$ $A \leftarrow P(A)$ $A \leftarrow A \oplus k_{h+1}$ $ciphertext \leftarrow A$

Decryption: Just the reverse.

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Random Permutation

- A block cipher is a permutation: $F_k: \{0,1\}^n \rightarrow \{0,1\}^n$.
	- \blacktriangleright Ideally it should be a truly random permutation.
	- \blacktriangleright How many bits do you need to represent a random permutation on n-bits?

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▶ Build a "random looking" permutation for large block-length from smaller random/random-looking permutations.

 \blacktriangleright Example:

- For $n = 64$, $F_k(x) = f_1(x_1)f_2(x_2)\cdots f_8(x_8)$ where the secret key k determines 8 random permutations on 8-bits.
- **In** Can you distinguish $F_k(x)$ from the output of a truly random permutation?

Mixing

\blacktriangleright The output bits of $F_k(x)$ are re-ordered/mixed. $\blacktriangleright x' \leftarrow F_k(x)$, then permute the bits of x' to obtain x_1 .

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- \blacktriangleright Recall one round of SPN \cdot
	- $A \leftarrow A \oplus k_i$ $A \leftarrow S(A)$ $A \leftarrow P(A)$

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- \blacktriangleright Recall one round of SPN \cdot
	- $A \leftarrow A \oplus k_i$ $A \leftarrow S(A)$
	- $A \leftarrow P(A)$
- \triangleright Obviously one round is insecure. (Recover the secret key given one input/output pair.)
- \triangleright Repeat the rounds several times. Hopefully small changes in input will have significant change in the output.

SPN Structure

- \blacktriangleright The smaller permutations f_i act as fixed substitution function or S-boxes.
- \blacktriangleright Unlike the Feistel Network, the S-boxes must be invertible in SPN.
- \triangleright Avalanche effect: small change in input should result in large change to the output.
- \triangleright SPN with several rounds achieve avalanche effect if
	- 1. S-boxes are so designed that changing a single input bit changes at least two bits in the output of the S-box.
	- 2. Output bits of any particular S-box is spread across different S-boxes in the next round by the mixing permutation.

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Advanced Encryption Standard

- \triangleright AES is an SPN but the permutation operation is replaced by two linear transformations (one of them is a permutation).
- \blacktriangleright All operations are byte oriented.
	- \blacktriangleright S-box maps 8-bits to 8-bits.
	- \blacktriangleright Allows efficient implementation on various platforms.
- \triangleright Block size of AES is 128-bits.
- \blacktriangleright Each round key is also 128-bits.
- \triangleright AES accepts three different key lengths and the number of rounds depends on the key length:
	- 1. 128-bit key: 10 rounds.
	- 2. 192-bit key: 12 rounds.
	- 3. 256-bit key: 14 rounds.

Performance

On Intel Xeon CPU E5-2698 v3 at 2.30GHz.

- \triangleright Several processors provide special instruction sets for AES, called AES-NI leading to significant speed-up.
- \triangleright AES-128-NI \approx 2400 MB/sec and AES-256-NI \approx 1800 MB/sec

Encrypting Large Messages

- ▶ You've a secure block cipher and a confidential message.
	- **If** Suppose the block cipher has a block length of n -bits and (for simplicity) the message length is some multiple of n , say, jn -bits.

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	- Divide the message into i blocks each of length *n*-bits.
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- If Is ECB a secure mode of encrypting your confidential data?

A picture is worth a thousand words!

Encryption by ECB mode [Source: Wikipedia].

Zoom at the beginning of the Pandemic used ECB for encryption! See: Move Fast and Roll Your Own Crypto: A Quick Look at the Confidentiality of Zoom Meetings [The Citizen Lab report, April 2020]

Cipher Block Chaining (CBC) Mode

Encrypt:

Start with an initialization vector $IV \in_R \{0,1\}^n$ and set $c_0 = IV$.

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- \triangleright Compute $c_i = E_k(m_i \oplus c_{i-1})$ for $1 \le i \le j$.
- ▶ Ciphertext: $c_0, c_1, ..., c_j$. Note: IV is sent in the clear.

Decrypt:

▶ Compute $m_i = E_k^{-1}$ $\overline{c_i}^{-1}(c_i) \oplus c_{i-1}$, for $1 \leq i \leq j$.

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Note:

- 1. The IV shouldn't be predictable.
- 2. Identical plaintexts with different IV results in different ciphertexts.
- 3. Any change in a plaintext block (m_i) affects c_i, c_{i+1}, \ldots
	- \triangleright Useful in the construction of a message authentication code (MAC).

Other Modes of Operation

- \triangleright Output feedback mode (OFB): An initialization vector IV is encrypted repeatedly using the block cipher to generate a key stream – actually a stream cipher.
- \triangleright Cipher feedback mode (CFB): Also a stream cipher.

$$
\begin{aligned} c_0 &= \mathsf{IV}.\\ z_i &= E_k(c_{i-1}). \end{aligned}
$$

- \blacktriangleright $c_i = m_i \oplus z_i$.
- \blacktriangleright Counter mode
	- ► Select a counter ctr and increment it as $T_i = ctr + i 1$ mod 2^n where n is the block length.

$$
\blacktriangleright z_i = E_k(T_i) \text{ and } c_i = m_i \oplus z_i.
$$

 \blacktriangleright Allows parallelism.

- 1. Video Lectures by Alfred Menezes: https://cryptography101.ca/crypto101-building-blocks
	- \triangleright Chapter V2 for Symmetric Key Encryption
- 2. Boneh and Shoup [draft]: Chapter 3 for Stream Cipher and Chapter 4 for Block Cipher.

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